P425/2
APPLIED
MATHEMATICS
Paper 2
Jul. / Aug. 2018
3 hours

''Together for Mathematics''

SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

SMATA JOINT MOCK EXAMINATIONS 2018

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** the ***eight*** questions in Section **A** and only **four** questions in Section **B**.

Any additional question(s) will **not** be marked.

All working must be shown clearly.

Begin each answer on a **fresh** sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Where necessary, take acceleration due to gravity, $\mathbf{g = 9.8\ ms^{-2}}$

Turn Over

SECTION A: (40 MARKS)

*Answer **all** questions in this Section*

1. A Continuous random variable has accumulative probability function given by

$$F(x) = \begin{cases} \log_2(x^k) & ; 0 \le x \le e \\ 1, & x \ge e \end{cases}$$

   (i) Show that k = 1n2.

   (ii) Obtain the p.d.f of X. ***(05 marks)***

2. A stone is projected from the top a cliff of height 25m with an initial speed of $12ms^{-1}$ at an angle of 60^0 to the vertical.
   Find the time it takes the stone to hit the sea-level. ***(05 marks)***

3. Given that $y = \sec 45^0 \pm 10\%$, find the limit within which the exact value of y lies. ***(05 marks)***

4. The table below shows the cost of ingredients used for making Chapats for two different birthday parties for 2015 and 2017.

Ingredients	**Cost**	
	2015	**2017**
Salt	200	350
Baking flour	3800	4600
Cooking oil	1500	1800

   By taking 2015 as a base year, Calculate the price relative for each ingredient and hence, obtain the average index number. ***(05 marks)***

5. A random variable X is such that X~ N (102, 16).
   Find P(| x – 100 | < 7.2). ***(05 marks)***

6. Given the information in the table below.

x	2.5	3.0	3.2	3.4	3.8
$f(x)$	1.56	1.82	1.95	2.05	2.13

   Use linear interpolation/Extrapolation to find

(i) $f(4.1)$

(ii) $f^{-1}(1.72)$ ***(05 marks)***

7. A hose pipe of cross section area 12cm^2 is located at a height of 6m above the ground, draws and issues water at a speed of 4.5ms^{-1}. Find the rate at which the water is oozing out of the pipe (Take density of water to be 1000kgm^{-3}) ***(05 marks)***

8. Find the centre of gravity of a uniform lamina whose shape is the area bounded by $y = x^2,$ the $x - axis$ and $x = 4$. ***(05 marks)***

SECTION B: (60 MARKS)

*Answer only **five** questions from this section.*

9. (a) Use the trapezoidal rule with five ordinates to evaluate

$$\int_{o}^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\, dx$$ to 3 decimal places.

(b) Find the exact value of $\int_{o}^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\, dx$ to 3 d.ps

(c) Find the absolute error in the function and state one way how this error can be reduced. ***(12 marks)***

10. Two equal forces each of magnitude P **N** have an angle between then 2α, if their resultant is twice that when the same forces have an angle between them as 2β,

(a) Prove that $8\,cos^2\beta$ - $\cos 2\,\alpha - 1 = 0$

(b) If $\propto = \frac{\pi}{4}$, find β. ***(12 marks)***

11. A random variable X has its p.d.f given by

$$P(x = x) = \begin{cases} \frac{k}{x}\, ; x = 1, 2, 3. \\ 0\, , otherwise \end{cases}$$

Find (i) value of Constant k

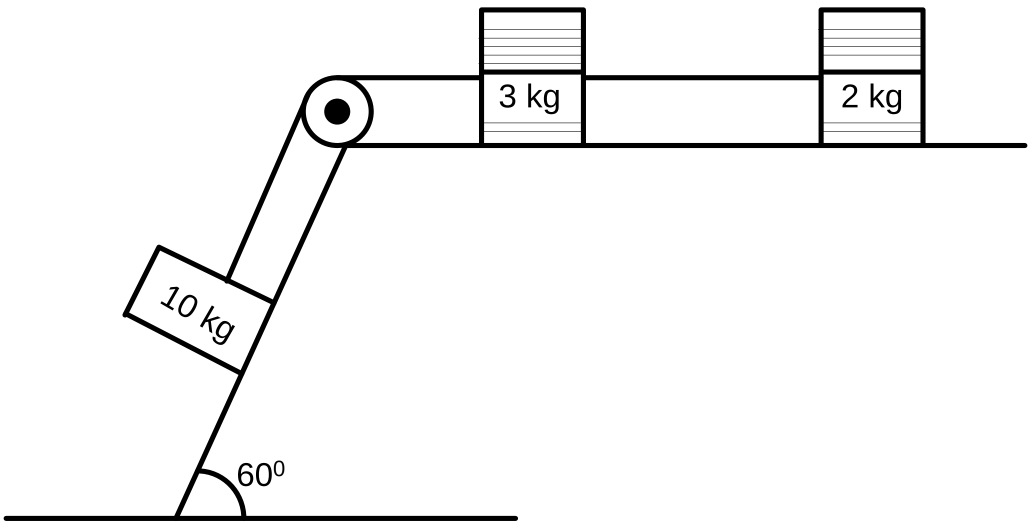
(ii) E $(x+1)^2$

(iii) Median

(iv) 3rd decile ***(12 marks)***

 Turn Over

12. The figure below shows a mass of 10kg placed on a smooth incline of inclination 60^0 attached to a mass of 3kg placed on a rough horizontal table by means of an inelastic string passing over a smooth pulley and connected to a second mass of 2kg on the table by means of another string. The coefficient of friction of the table is $\frac{1}{3}$.



(a) Find the acceleration of each mass.

(a) Find the tension in each string.

(b) Reaction on the pulley. ***(12 marks)***

13. Annet stays in Kenya and Bob stays in Uganda. The probability that Annet will go to china in December this year is $\frac{3}{5}$ and that of Bob is $\frac{3}{8}$.

(a) Find the probability that they are likely to be in different countries next year.

(b) The probability that patience passes Biology, Chemistry and Mathematics is 0.7, 0.8 and 0.65 respectively.

(i) Find the probability that she passes at most one subject.

(ii) If we know that she passed at most one subject. What is the probability that she passed Mathematics.

(12 marks)

14. (a) Construct a flow chart that computes and prints the average of the squares of the first six counting numbers. Perform a dry run for your flow chart.

(b) Locate graphically the positive root of the equation $e^{-x} = 4 - 3x$ and hence, use linear interpolation to find the root of the equation to 2 decimal places. ***(12 marks)***

15. (a) A cyclist A appears to be moving at a velocity of $10ms^{-1}$ on a bearing of 330^o to a cyclist B moving with a velocity of $\sqrt{8}\ ms^{-1}$ on a bearing of 045^o. Find the true velocity of the cyclist .

(b) A particle of mass 2kg, moves on a space curve accelerating at a rate of $\mathbf{a} = (\cos 2\ t\,\boldsymbol{i} + \ \sin 2\mathbf{j} + t\boldsymbol{k}\,)$. Find the power developed after 3 seconds. ***(12 marks)***

16. The table below shows height in centimetres of 25 students in a certain school.

Height (cm)	< 10	< 20	< 25	< 30	< 50	<55	< 65
Number of students	0	3	7	15	17	23	25

Calculate

(i) Mean height

(ii) Variance

(iii) Mode

(iv) Middle 70% of the height. ***(12 marks)***

END

MARKING GUIDE FOR P425/2 MOCK 2018

SMATA MOCK EXAM, 2018

SECTION A.

① (i) $F(x) = \begin{cases} \log_2 x^e, & 0 \le x \le e \\ 1, & x \geq e. \end{cases}$, but $F(e) = 1$

$F(e) = \frac{K \ln e}{\ln 2}$ M1

$\Rightarrow 1 = \frac{K \ln e}{\ln 2}$

$1 = \frac{K}{\ln 2}$, $K = \ln 2$ B1

(ii) $f(x) = \frac{d}{dx}\left(\frac{K \ln x}{\ln 2}\right) = \frac{K}{x \ln 2} = \frac{1}{x}$ B1, $\frac{d}{dx}(1) = 0$ B1

$f(x) = \begin{cases} \frac{1}{x}, & 0 \le x \le e \\ 0, & \text{elsewhere} \end{cases}$ A1

05

② Using $y = ut + \frac{1}{2}at^2$

$-25 = 12\sin 30 t - \frac{1}{2}gt^2$

$-25 = 6t - 4.9t^2$ B1

$4.9t^2 - 6t - 25 = 0$ M1 B1

$t = 2.953$ or $t = -1.728$

time $= 2.953$ s A1

05

③ $e_\theta = 0.1^\circ = 0.001745$ B1

$y = \sec 45 \pm 10\%$

$y_{max} = \sec 45.1^\circ = 1.4167$ B1

$y_{min} = \sec 44.9^\circ = 1.4118$ B1

limit $= 1.4118 \le$ Exact value < 1.4167 A1 M1

05

④

Ingradeent	P_0 cost 2015	P_1 2017	$P_r = \left(\frac{P_1}{P_0} \times 100\right)$
salt	200	350	175 B1
B.flour	3800	4600	121.1 B1
c.oil	1500	1800	120 B1

$\Sigma P_r = 416.1$

$A.I.N = \frac{\Sigma P_r}{r} = \frac{416.1}{3}$ M1 $= 138.7$ A1

05

(5). $X \sim N(102, 16)$

$\sigma^2 = 16 \qquad \mu = 102$

$\sigma = 4$ B1

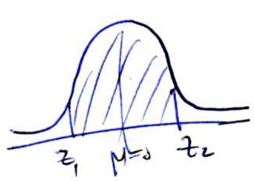
$P(|X - 100| < 7.2$

$= P(100 - 7.2 < X < 100 + 7.2)$ M1

$= P(92.8 \leq X \leq 107.2)$

$= P\left(\frac{92.8 - 102}{4} \leq Z < \frac{107.2 - 102}{4}\right)$

$= P(-2.3 \leq Z \leq 1.3)$ B1



$= \phi(2.3) + \phi(1.3)$

$= 0.48928 + 0.4032$ M1 B1

$= 0.89248$ A1

05

(6) (i)

x	3.4	3.8	4.1
$f(x)$	2.05	2.13	y

B1

$\frac{y - 2.13}{4.1 - 3.8} = \frac{2.13 - 2.05}{3.8 - 3.4}$ M1

$y = 2.13 + \frac{(2.13 - 2.05)(4.1 - 3.8)}{3.8 - 3.4} = 2.19$ A1

(ii)

x	2.5	x	3.0
$f(x)$	1.56	1.72	1.82

$\frac{x - 2.5}{1.72 - 1.56} = \frac{3.0 - 2.5}{1.82 - 1.56}$ M1

$x = 2.5 + \frac{(3.0 - 2.5)(1.72 - 1.56)}{1.82 - 1.56}$

$= 2.8077$ A1

05

(7). $P = k.e/s + p.e/s$

$= \frac{1}{2} m_s v^2 + m_s g h$

but volume rate $= \left(\frac{12}{100^2} \times 4.5\right) = 0.0054\ m^3 s^{-1}$

mass rate $=$ volume rate $\times$ density of water

$= 0.0054 \times 1000$ B1

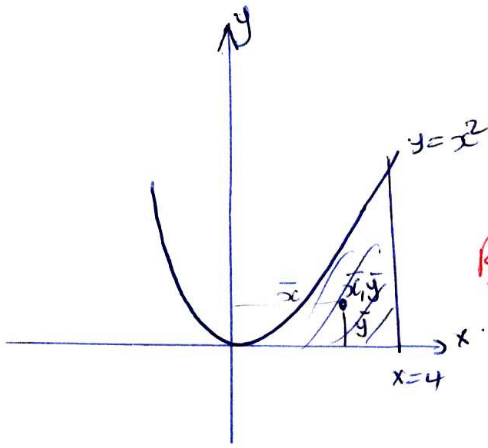
$= 5.4\ kgs^{-1}$

$= \frac{1}{2} \times 5.4 \times 4.5^2 + 5.4 \times 9.8 \times 6$ B1 B1 M1

$= 372.195\ Js^{-1}$ A1

05

8



B1

Let the weight per unit area be ρ.

taking moments about the origin,

$$\begin{matrix}\rightarrow \\ \uparrow\end{matrix}\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} \rho \int_0^4 y\,dx = \rho \begin{pmatrix} \int_0^4 xy\,dx \\ \int_0^4 \frac{1}{2}y^2\,dx \end{pmatrix}$$ M1

$$\rightarrow \quad \bar{x} \rho \int_0^4 x^2\,dx = \rho \int_0^4 x^3\,dx$$

$$\bar{x} = \frac{\int_0^4 x^3\,dx}{\int_0^4 x^2\,dx} = \frac{\frac{1}{4}x^4 \Big|_0^4}{\frac{1}{3}x^3 \Big|_0^4} = \frac{64}{\frac{1}{3}(64)} = 3$$ B1

$$\uparrow \quad \bar{y} \rho \int_0^4 y\,dx = \rho \int_0^4 \tfrac{1}{2}y^2\,dx.$$

$$\bar{y} \rho \int_0^4 x^2\,dx = \rho \int_0^4 \tfrac{1}{2}x^4\,dx$$

$$\bar{y} = \frac{\int_0^4 \frac{1}{2}x^4\,dx}{\int_0^4 x^2\,dx} = \frac{\frac{1}{2}\times\frac{1}{5}x^5 \Big|_0^4}{\frac{1}{3}x^3 \Big|_0^4} = \frac{\frac{1}{10}(1024)}{\frac{64}{3}}$$

$$\bar{y} = \frac{1}{2}\times\frac{48}{5} = \frac{24}{5}$$ B1

$$(\bar{x}, \bar{y}) = \left(3, \frac{24}{5}\right)$$ A1

05

No. 9 (a)

$h = \frac{\frac{\pi}{4} - 0}{4}$ B1 $= \frac{\pi}{16}$

x	$f(x) = \frac{2}{\sqrt{1-x^2}}$	
0	2.00	
$\frac{\pi}{16}$		2.0397
$\frac{2\pi}{16}$		2.1747
$\frac{3\pi}{16}$		2.4749
$\frac{4\pi}{16}$	3.2311	
T B1	5.2311	6.6893 B1

$A \simeq \frac{1}{2}h\left[f_0 + f_{n+1} + 2(f_1 + f_2 + \dots + f_{n-1})\right]$

$A \simeq \frac{1}{2}\left(\frac{\pi}{16}\right)\left[5.2311 + 2(6.6893)\right]$ M1

$A \simeq 1.8270$

$A \simeq 1.827$ (3dps) A1

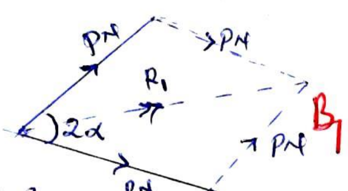
9(b) $\int_0^{\frac{\pi}{4}} \frac{2}{\sqrt{1-x^2}}\,dx = 2\sin^{-1}(x)\Big|_0^{\frac{\pi}{4}}$ B1 $= \left[2\sin^{-1}\left(\frac{\pi}{4}\right) - 2\sin^{-1}(0)\right] = 1.807$ A1 M1B1

(c) $e_{abs} = \left|\left(\text{Exact value}\right) - \text{approximate value}\right| = |1.807 - 1.827| = 0.02$ MA1 $\frac{}{12}$

This error can be reduced by (i) Increasing number of strips B1
(ii) Increasing number of d.ps $\frac{}{12}$

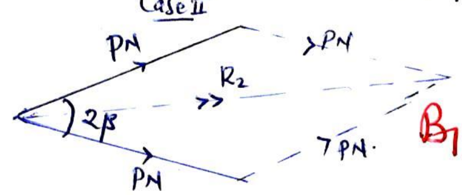
10 Case I

(a)



B1 $R_1 = 2R_2$

Case II



B1

$R_1^2 = p^2 + p^2 + 2p^2\cos 2\alpha$ B1

$R_2^2 = p^2 + p^2 + 2p^2\cos 2\beta$ B1

$R_1^2 = 4R_2^2$

$2p^2 + 2p^2\cos 2\alpha = 4(2p^2 + 2p^2\cos 2\beta)$ M1

$1 + \cos 2\alpha = 4 + 4(2\cos^2\beta - 1)$ B1

$8\cos^2\beta - \cos 2\alpha + 4 - 4 - 1 = 0$ B1

$8\cos^2\beta - \cos 2\alpha - 1 = 0$ B1

(b) $\alpha = \frac{\pi}{4}$

(b) $\alpha = \frac{\pi}{4}$

$8\cos^2\beta - \cos\frac{\pi}{2} = 1$ M1

$\cos^2\beta = \frac{1}{8}$

$\beta = \cos^{-1}\left(\pm\frac{1}{\sqrt{8}}\right)$ B1

$\beta = 69.3°$ or A1

and $\beta = 110.7°$ A1 $\frac{}{12}$

11. (a)

(i) $\sum_{\text{all } x} P(X=x) = 1$, but $P(X=x) = \begin{cases} \frac{k}{x}, & x=1,2,3 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow \frac{k}{1} + \frac{k}{2} + \frac{k}{3} = 1$ M1 B1

$k\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right) = 1$

$k\left(\frac{11}{6}\right) = 1$ B1

$k = \frac{6}{11}$ A1 $\therefore P(X=x) = \begin{cases} \frac{6}{11x}, & x=1,2,3 \\ 0, & \text{otherwise} \end{cases}$

(ii) $E(X+1)^2 = E(X^2+2X+1)$

$= E(X^2) + 2E(X) + 1$ M1

$E(X^2) = 1^2\left(\frac{6}{11}\right) + 2^2\left(\frac{6}{22}\right) + 3^2\left(\frac{6}{33}\right) = \frac{6}{11} + \frac{12}{11} + \frac{18}{11}$ B1

$= \frac{36}{11}$ ~~A1~~

$E(X) = 1\left(\frac{6}{11}\right) + 2\left(\frac{6}{22}\right) + 3\left(\frac{6}{33}\right) = \frac{6}{11} + \frac{6}{11} + \frac{6}{11} = \frac{18}{11}$ B1

$E(X+1)^2 = \frac{36}{11} + 2\left(\frac{18}{11}\right) + 1 = \frac{83}{11}$ B1 A1

(iii)

x	1	2	3
$P(X=x)$	$\frac{6}{11}$	$\frac{6}{22}$	$\frac{6}{33}$
$F(x)$	$\frac{6}{11}$	$\frac{9}{11}$	1

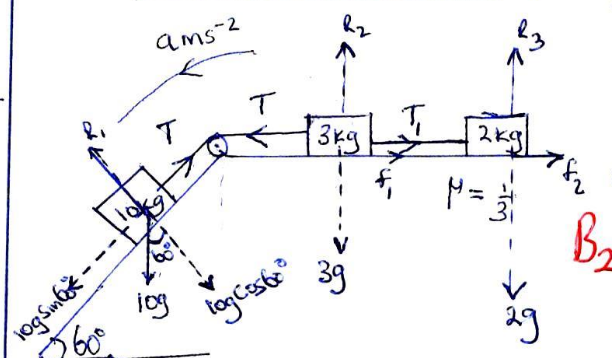
B1

Median = 1 A1

3rd decile = 1 A1

12

12



B2

a) $F = ma$;

10kg, $10g\sin 60° - T = 10a$

$5g\sqrt{3} - T = 10a$ — (1) B1

3kg, $T - T_1 - f_1 = 3a$

$T - T_1 - g = 3a$ — (2) B1

2kg, $T_1 - f_2 = 2a$

$T_1 - \frac{2}{3}g = 2a$ — (3) B1

Solving gives

$5g\sqrt{3} - 10a - 2a - \frac{2}{3}g - g = 3a$ M1

$a = \frac{6.9936 \times 9.8}{15} = 4.5691\,ms^{-2}$ A1

b) $T_1 = 2(4.574) + \frac{2}{3}g = 15.6715N$ M1 A1

$T = 5\sqrt{3}g - 10(4.574) = 39.1795N$ M1 A1

c) $R = T_1 = 39.1795N$ M1 A1

12

No. 13(a)

(i) $P(A) = \frac{3}{5}$, $P(A') = \frac{2}{5}$

$P(B) = \frac{3}{8}$, $P(B') = \frac{5}{8}$

$P(\text{Different Countries}) = P(A' \cap B') + P(A \cap B') + P(A' \cap B)$

$= \left(\frac{2}{5} \times \frac{5}{8}\right) + \left(\frac{3}{5} \times \frac{5}{8}\right) + \left(\frac{2}{5} \times \frac{3}{8}\right)$ B1 B1 B1 M1

$= \frac{31}{40}$ A1

(b) $P(B) = 0.7$, $P(B') = 0.3$

$P(C) = 0.8$, $P(C') = 0.2$

$P(M) = 0.65$, $P(M') = 0.35$

(i) $P(\text{Atmost one}) = P(B \cap C' \cap M') + P(B' \cap C \cap M') + P(B' \cap C' \cap M) + P(B' \cap C' \cap M')$

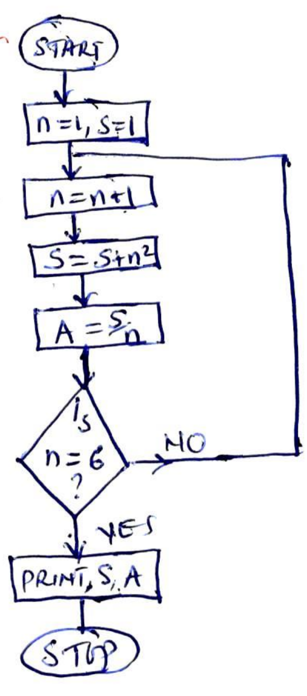
$= (0.7 \times 0.2 \times 0.35) + (0.3 \times 0.8 \times 0.35) + (0.3 \times 0.2 \times 0.65) + (0.3 \times 0.2 \times 0.35)$ B1 B1 B1 B1

$P(T) = 0.193$ A1

(ii) $P\left(\frac{M}{T}\right) = \frac{P(M \cap T)}{P(T)} = \frac{P(B' \cap C' \cap M)}{P(T)} = \frac{0.3 \times 0.2 \times 0.65}{(0.193)}$ M1 $= \frac{39}{193}$ A1

12

(14)(a)



B1 B1 B1

The dry run

n	S	A
1	1	1
2	5	2.5
3	14	14/3
4	30	7.5
5	55	11
6	91	91/6

B1 B1 B1

(b)(i) $f(x) = e^{-x} + 3x - 4$

x	0	1	2
y	-3	-0.6	2.1

B1

(ii) $x_0 = 1.2$

$f(1.2) = e^{-1.2} + 3(1.2) - 4 = -0.0988$

1.2	x_1	2
-0.099	0	2.135

B1

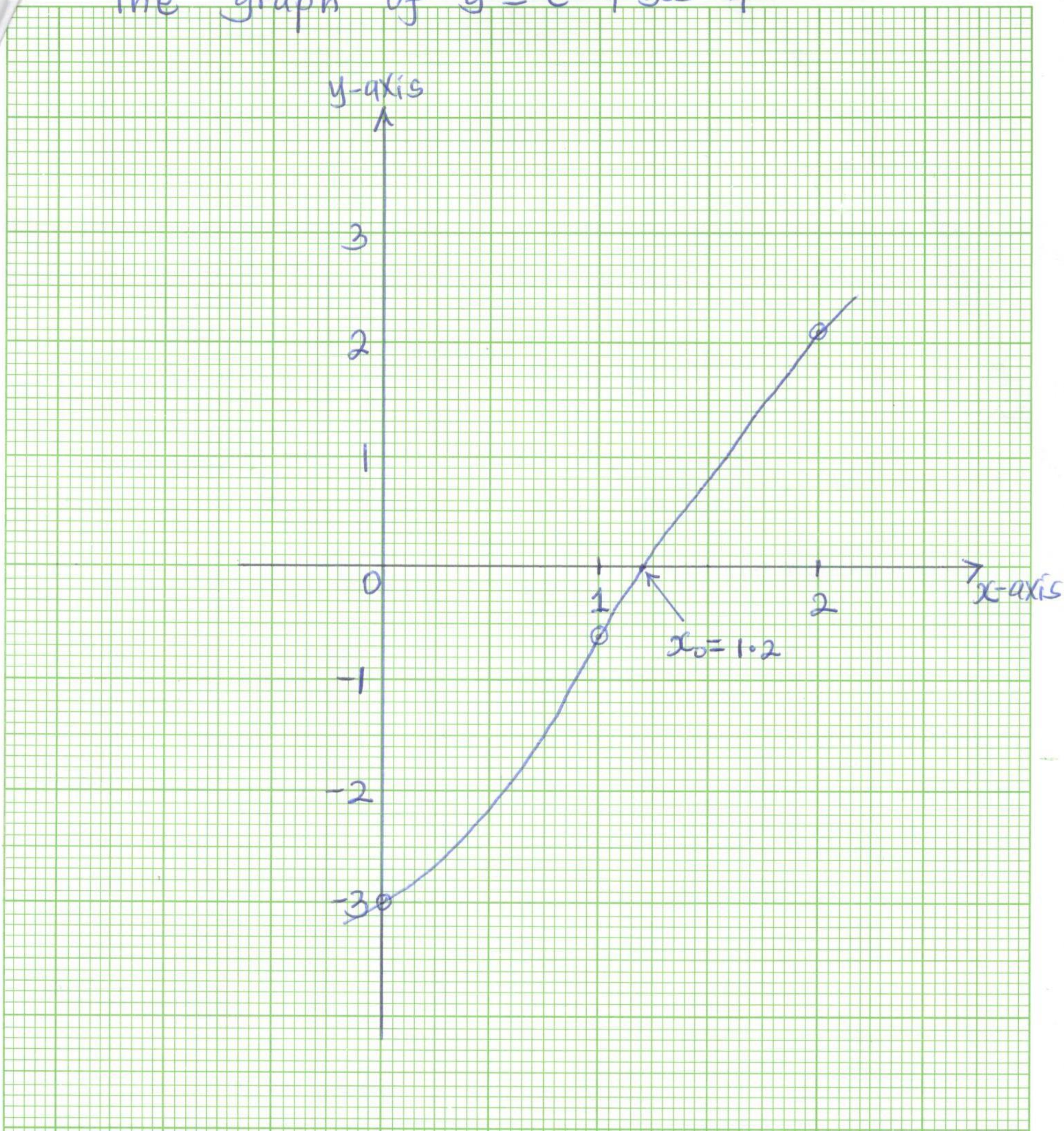
$\frac{2 - 1.2}{2.135 - {}^{-}0.099} = \frac{x_1 - 1.2}{0 - {}^{-}0.099}$ M1

$x_1 = 1.24$ A1

12

No-14

The graph of $y = e^{-x} + 3x - 4$



15(a) ${}_A V_B = \begin{pmatrix} -10\cos 60^\circ \\ 10\sin 60^\circ \end{pmatrix} = \begin{pmatrix} -5 \\ 5\sqrt{3} \end{pmatrix}$ B1

$V_B = \begin{pmatrix} \sqrt{8}\cos 45^\circ \\ \sqrt{8}\sin 45^\circ \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ B1

But ${}_A V_B = V_A - V_B$

~~$V_A = {}_A V_B$~~ $V_A = {}_A V_B + V_B = \begin{pmatrix} -5 \\ 5\sqrt{3} \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 2+5\sqrt{3} \end{pmatrix} = \begin{pmatrix} -3 \\ 10.66 \end{pmatrix}$

B1 B1 M1 A1

b) $P = \int m\, a \cdot a\, dt$, but $a \cdot a = \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} \cdot \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} = \cos^2 2t + \sin^2 2t + t^2$ M1 B1 M1 $= 1+t^2$ B1

$P = \int_0^3 2(1+t^2)\,dt = \int_0^3 2 + 2t^2\,dt = 2t + \frac{2}{3}t^3 \Big|_0^3 = 24\text{W}$ M1 A1

12

16.

class boundary	class	F	f	i	$f.d = \frac{f}{i}$	x	x^2	fx	fx^2
		0	0						
<10	0 − 10	3	3	10	0.3	15	225	45	675
<20	10 − 20	7	4	5	0.8	22.5	506.25	90	2025
<25	20 − 25	15	8	5	1.6	27.5	756.25	220	6050
<30	25 − 30	17	2	20	0.1	40	1600	80	3200
<50	30 − 50	23	6	5	1.2	52.5	2756.25	315	16537.5
<55	50 − 55	25	2	10	0.2	60	3600	120	7200
<65	55 − 65		$\Sigma f=25$					$\Sigma fx = 870$	$\Sigma fx^2 = 35687.5$

B1 B1 B1

i) Mean height $= \frac{\Sigma fx}{\Sigma f} = \frac{870}{25} = 34.8$cm M1 A1

ii) Variance $= \frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2 = \frac{35687.5}{25} - (34.8)^2 = 216.46$ M1 A1

iii) Mode $= L_1 + \left(\frac{d_1}{d_1+d_2}\right)i = 25 + \left(\frac{0.8}{0.8+1.5}\right)\times 5 = 26.7391$cm M1 A1

iv) 0 — 15 — 70 — 15 — 100

$P_{15} = L_b + \left(\frac{\frac{15}{100}\Sigma f - cf_b}{f_m}\right)i = 20 + \frac{(3.75-3)\times 5}{4} = 12.5$ B1

$P_{85} = L_b + \left(\frac{\frac{85}{100}\Sigma f - cf_b}{f_m}\right)i = 50 + \left(\frac{21.25-17}{6}\right)\times 5 = 53.5417$ B1

Middle 70% $= P_{85} - P_{15} = 53.5417 - 12.5$

$= 41.0417$cm A1

12